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Dim Object Detection and Characterization Through Multi-frame Imaging

Szymon Gladysz

**European Southern Observatory
Karl-Schwarzschild-Strasse 2
Garching bei Munchen, Germany D85748**

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Asher Space Research Institute
Technion - Israel Institute of Technology
Haifa, Israel

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“Dim object detection and characterization through multi-frame imaging”

Final report

Szymon Gładysz (Principal Investigator) with:

Natalia Yaitskova (European Organization for Astronomical Research in the Southern Hemisphere)

Julian Christou (Gemini Observatory)

Roberto Baena Gallé (University of Barcelona)

Rao Gudimetla (Air Force Research Laboratory)

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1. Summary

In this document we report on activities related to the project “Dim object detection and characterization through multi-frame imaging”, grant number FA8655-09-1-3052, in the second year of funding. We have made a significant breakthrough in our attempts to explain theoretically peculiar behaviour of adaptive-optics (AO) speckle, and subsequently we developed a theory applicable to all frequently-encountered types of speckle, with or without AO, occurring on small-, or large-diameter telescopes, and for small or significant optical aberrations.

We also continued the work on image reconstruction after AO. We have tested and published results of accuracy of five algorithms. The results show, among other things, that the most-widely-adopted approach produces the worst results. More importantly, we have developed a methodology for testing deconvolution codes for the problem of measuring relative flux ratio between space objects.

Our work so far has resulted in one journal paper and six papers published in conference proceedings. Additionally, one more journal paper has come back for minor revisions, one journal paper was recently submitted, and one paper was submitted to a conference for consideration as an oral presentation.

2. Introduction

Speckle noise is a ubiquitous manifestation of roughness of surfaces being illuminated with coherent light, e.g. with laser. Briefly: light reflected from a diffuser with roughness comparable to the illuminating wavelength creates a characteristic granular pattern on the screen, i.e. a speckle image (Fig. 1 left). Speckle noise is a problem in laser optics, in imaging through turbulence where Earth’s atmosphere is the scattering medium, and in retinal imaging, to name just a few fields in optics. The effect of speckle on imaging through turbulence is illustrated in Fig. 1.

Speckle is a random phenomenon and therefore it is best analyzed statistically. “Pure” speckle can be pictured in terms of the random walk. Its properties are very well understood by now. Also, some special cases have been analyzed when the number of steps in the random walk is small, and/or when the direction of the walk is constrained [1]. Nevertheless, the statistical properties of speckle have been assumed to be spatially-stationary, i.e. speckles were assumed to behave similarly independently of the position in the image plane.

In the era of high-precision and/or adaptive optics speckle noise has been observed to be no longer spatially-stationary. The point-spread function (PSF) for a well-corrected system demonstrates a complicated morphology with a central peak, Airy rings, pinned speckles on top of the rings, and an elongated halo due to the uncorrected aberrations.

There is a great interest in the statistical properties of “corrected”, spatially-non-stationary speckles [2-4]. The central application in astronomy is high-contrast imaging of faint companions to stars. For this application one has to use so-called extreme AO which yields almost flat wavefronts and almost static diffraction-limited PSF. The difference between “almost flat” and “perfectly flat” gives rise to speckles. The statistical behavior of these speckles has been modeled using well established tools and classic assumptions [4] but this theory failed to predict non-stationarity of speckles and the peculiar behavior of the central peak.

We have developed a complete theoretical framework for speckles which makes only one, classic assumption about the nature of the aberration: that it can be decomposed into a finite number of “cells” or “patches”. The framework is very general in that it makes no assumptions about the number of these patches (previous theories assumed large numbers and made use of the Central Limit Theorem to avail of the Gaussian statistics) and no assumptions about

the statistical distribution of the values of the cells which could be Gaussian, uniform, gamma, etc. This framework is fully back-compatible with the classic model of Goodman [1] but it also explains all of the interesting phenomena recently encountered in AO experiments.

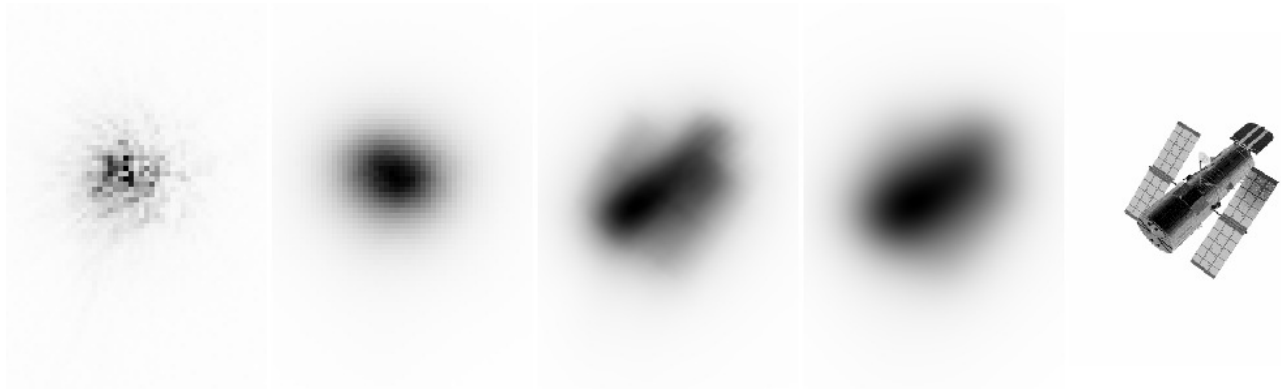


Fig. 1. Illustration of image quality degradation in the presence of turbulence between the object and the observer: (from left to right) speckle image (from the 2.4m telescope, Magdalena Ridge Observatory), long-exposure image from the same telescope, schematic image of the Hubble Space Telescope convolved with the speckle image from the leftmost panel, same image convolved with the long exposure, model of the Hubble Space Telescope used in the simulation. All images displayed on an inverted linear scale.

Another theme of this research is using theoretical information about statistics of intensity in AO imagery to enhance the capabilities of existing AO systems without extra cost. The simplest – albeit the most expensive – way to improve AO correction is to buy a more expensive deformable mirror having more degrees of freedom, and therefore capable of better aberration compensation. Instead we are exploring gains brought upon by image post-processing.

3. Methods, Assumptions, and Procedures

3.1 Theory: first order speckle statistics for arbitrary aberration strength

Drs. Gladysz and Gudimetla have worked on statistics of the real and imaginary parts of electric field in the focal plane of an imaging system. Finite number of wavefront cells was assumed. Additionally, the geometry of the cells was assumed to be periodic as in a diffraction grating. The goal was to derive mean and variance of the real and imaginary parts and check for possible non-stationarity, i.e. change in the moments across the focal plane. Stationarity is the fundamental assumption of the theories invoking the modified Rician distribution to model the statistical behavior of AO speckles [3,4]. Another fundamental assumption of these theories is great number of wavefront cells, such that the real and imaginary parts of the resulting phasor in the focal plane follow Gaussian statistics. We chose not to be constrained by that assumption either. Subsequently, we have shown that the diffraction-grating model for the highly-corrected wavefront results in non-stationary statistics for ALL of the investigated moments. Specifically, the widely accepted – although never tested – assumption that the variances of the real and imaginary parts are equal was shown not to be true even for a single point in the image plane. Furthermore, we have shown that one cannot assume a great number of wavefront cells and use the Rician distribution to model the behavior of

the central peak of the PSF. After plugging of the derived moments into the joint Gaussian probability density function of the real and imaginary parts and numerical integration to obtain the distribution of intensity we found no trace of the negative skewness which was observed in experiments and explained by us with an earlier, simpler model [2]. In the future results of this work will be submitted to Optics Letters.

Concurrently, Drs. Yaitskova and Gladysz continued to develop a general theory of the first order statistics of speckle intensity. They investigated the behavior of mean and variance of intensity using, again, a diffraction-grating model for the wavefront. Two examples of the validity of such a model are the primary mirror of a segmented telescope [5] and the residual phase after high-order AO correction, both presented in Fig. 2. To derive general equations nothing was assumed regarding the strength of the aberrations: the standard deviation of the wavefront phase was a free parameter. Hence, this analysis is relevant for both: strong and weak aberrations. Also the number of cells was a free parameter because the analysis was not to be restricted to the Gaussian speckle only.

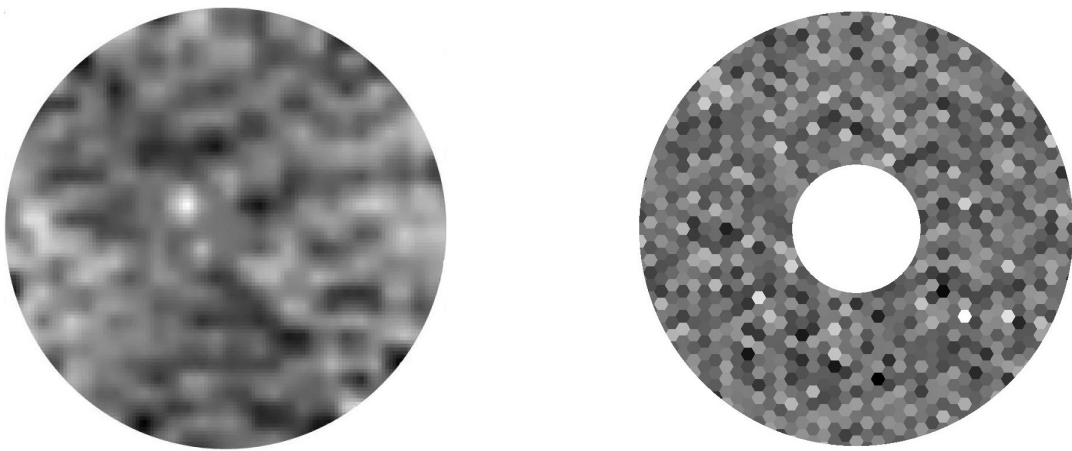


Fig. 2. (Left) A reconstructed wavefront from the High Order Testbench after extreme AO compensation. (Right) Pupil of the European Extremely Large Telescope composed of 894 hexagonal segments.

3.2 Comparison of image reconstruction approaches

With Drs. Julian Christou (Gemini Observatory), Lewis Roberts (CALTECH) and Jack Drummond (AFRL) we have set up a group of researchers interested in image reconstruction after AO. We first decided to carry out a blind test of reconstruction methods for the simple case of a two-point object. Relative intensity between the two points was varied, the effect of atmospheric turbulence was simulated and the test data was distributed among the collaborators. One of the tested methods called PDF deconvolution is a “spin-off” of our theoretical work on speckle statistics. In this method traditional 2-D image deconvolution is replaced by a 1-D time-series deconvolution. The algorithm makes use of our models for on-axis and off-axis intensity statistics. The other methods were: multi-frame blind deconvolution (MFBD) code called IDAC, single-frame PSF fitting, and a new approach combining maximum-likelihood with wavelets (AWMLE).

Concurrently, we have applied three deconvolution methods to simulated images of the Hubble Space Telescope. The tested algorithms were: maximum-likelihood with wavelets or curvelets (AWMLE), MFBD, and myopic, single-frame deconvolution code called MISTRAL [6]. MISTRAL differs from the

other two methods in that it uses regularization, i.e. some user-specified assumption about the object being imaged.

4. Results and Discussion

4.1 Theory: first order speckle statistics for arbitrary aberration strength

Fraunhofer diffraction integrals were used throughout the analysis and we believe it was the first time that Fourier optics methods have been employed together with random wavefront variations to explain speckle statistics. This “marriage” turned out to be very successful.

The theory predicts that:

- The Rician model is only accurate for very small aberrations and for off-axis speckles.
- The Rician model cannot explain statistics of on-axis intensity for well-corrected wavefronts.
- The existence of “pinned” speckles (speckles amplified by the Airy pattern) which have been observed experimentally is explained within the model, for small phase aberrations.
- The variance of the Strehl ratio (normalized on-axis intensity) is a complicated function of the wavefront phase variance (Fig. 3). Specifically, maximum variability of the Strehl ratio occurs for phase variance approximately equal to one. This is counterintuitive: most researchers believe that increasing aberrations will always increase the variability of the central peak. But even a heuristic explanation points to the contrary: the variability of the peak cannot be driven to infinity by increasing aberrations; this variance has to be transferred to the other speckles, and in the end – for fully-developed speckle pattern – all speckles must have their variance equal to their mean value. And so, decreasing the mean value of the central peak cannot be accompanied by a simultaneous and monotonic increase in its variance because in the end – for very large aberrations – their values are tied together.

From Fig. 3 one can also see why the early attempts to use the modified Rician distribution to model statistics of the on-axis intensity were partly successful. In the range of phase standard deviations from 0 to approximately 1.8rad the Rician model does not work. But above 1.8rad the curves coincide and the Rician model can be applied to describe the standard deviation of the Strehl ratio for relatively large aberrations. This transition point of 1.8rad is virtually insensitive to the number of cells. Therefore we have shown that the value of 1.8 radians is a limit where the Rician approximation for the image center is still applicable.

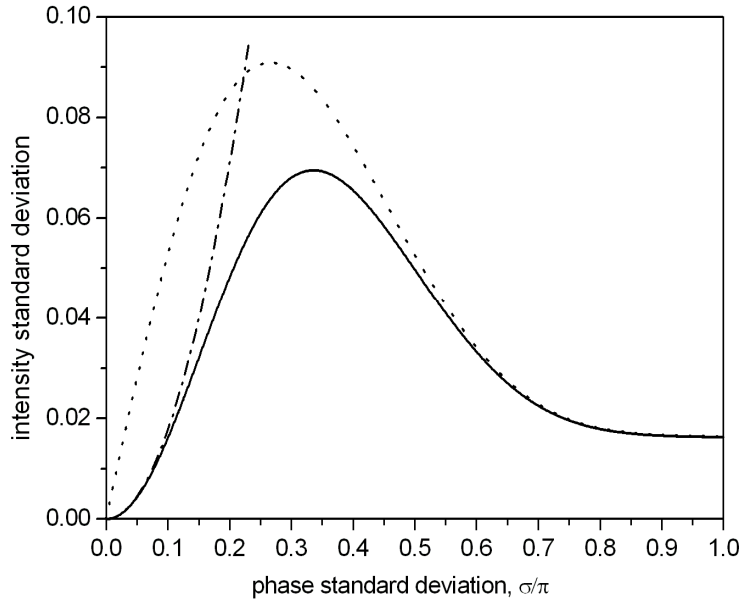


Fig. 3. Standard deviation of central peak intensity. Solid: exact curve, dash-dotted: quadratic approximation, dotted: standard deviation calculated from the Rician formula.

The paper describing our analysis is submitted to Journal of the Optical Society of America. Our work on the problem of non-spatially-stationary speckle has spurred other ideas which we plan to develop in the future:

- Since now we have the tools to predict statistics of speckles with or without AO we will test whether injecting this theoretical prediction about speckle variability into the process of (multi-frame) image restoration would lead to higher spatial resolution of reconstructed images of space objects.
- The second point is linked to the first one. Goodman [1] argues that statistics of the (squared) short-exposure optical transfer function (OTF) can be approximated by the statistics of intensity. Image spectrum signal-to-noise ratio, defined as the ratio of expected value of the OTF to its standard deviation, is a popular performance metric in speckle imaging [7]. Analytical prediction of this metric for AO, based on models of compensated structure functions, is rather complex. Our image-domain analysis could provide an alternative. Moreover, we strongly believe a full statistical distribution, and not just its moments, can be derived. This distribution could be used as a “prior”, a statistical constraint for the multi-frame blind deconvolution [8]. This would accompany the intensity constraint mentioned in the first point.
- Since our theoretical model has only two parameters: number of wavefront cells and phase variance, it can be seen that the statistics of speckle pattern carry information about the structure of the scattering medium. This is not a new idea as speckle interferometry is a well developed area by now. But, to our knowledge there have been no attempts in the AO community to join the partially-developed speckle model and the theory of turbulence to measure the spatial statistics of wavefronts travelling through atmospheric turbulence. We plan to test whether the measurements of speckle contrast could give information about the coherence length of the atmosphere. If successful, this idea could pave the way for a new type of turbulence measurement, not just for astronomy but also for ophthalmic applications

(where the applicability of the atmospheric models is questionable at best, but the most general, cell-based representation of the phase is still valid and so is our speckle theory).

- Finally, we have the starting point to explore the statistics of speckle in non-stationary turbulence (where wind speed and optical coherence length are changing). The statistics will be obtained by making use of the models of the probability density functions (PDF) of coherence length and wind speed. Residual phase variance after AO can be related to these two non-stationary quantities via error budget equations. Integrating the speckle moments' equations with respect to coherence length and/or wind speed will yield the expected value of these moments after all “allowed” turbulent states. We will check whether speckle contrast increases or decreases when moving from stationary to non-stationary turbulence.

4.2 Image reconstruction after AO

In our tests of image reconstruction for a simple object containing two delta functions the final performance metric we chose to compare the methods was the error in relative photometry computed for the two points in the deconvolved image. In Fig. 4 this metric is plotted for the case of PSF well matched to the observations (Strehl ratio difference of only 2%) and mismatched PSF (difference of 6-7%).

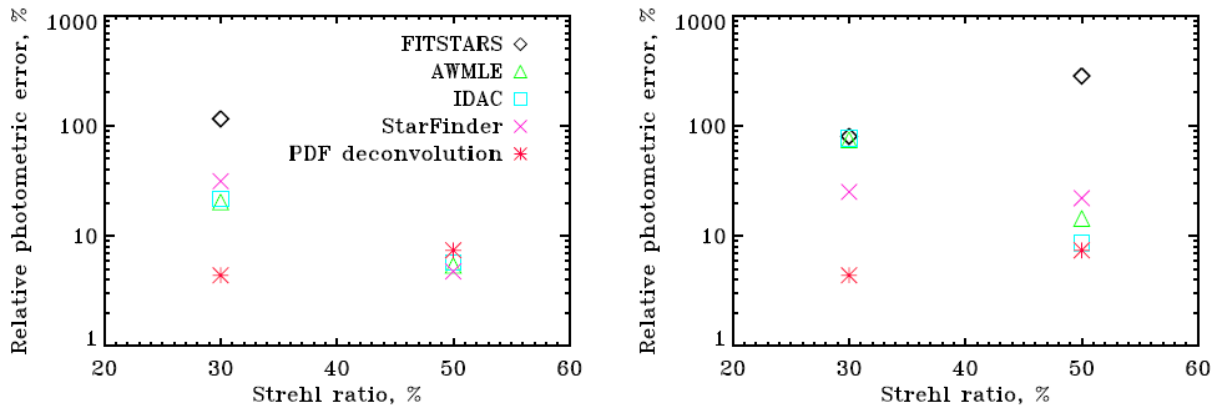


Fig. 4. Mean photometric error for the five tested algorithms. Left: PSF well-matched to the observations. Right: mismatched PSF. The results for PDF deconvolution are identical in both panels because this method does not rely on a PSF estimate. Error value for FITSTARS in the 50% SR, matched-PSF case was very high ($\sim 10\,000$) and we omitted it from the plot in order to have the y-axis scale which better shows differences between the other methods.

Interestingly, it can be seen in Fig. 4 that the most-widely-adopted approach that is FITSTARS produces the worst results for these simulated datasets. FITSTARS was developed with binary stars in mind and therefore its poor photometric precision is particularly worrying. These results show the need for testing of the codes. This work was presented as a talk at the AMOS conference 2010.

Another set of tests was carried out on the images of the extended source (Hubble Space Telescope, Fig. 1). Here, performance metric was the visual quality of the reconstructed image. We wanted to check whether the widely-accepted assumption that static-PSF codes do not work well with AO data is true. To make the comparison fair and as close as possible to the imaging scenarios which could be employed in real life we

used twenty noisy frames (10 counts rms readout noise per frame) for MFBD, two integrated noisy frames (10 counts rms readout noise for either image composed of ten frames) for AWMLE, and one integrated image with one realization of readout noise for MISTRAL. The input data is shown in Fig. 5 and Fig. 6 shows the reconstructed images.

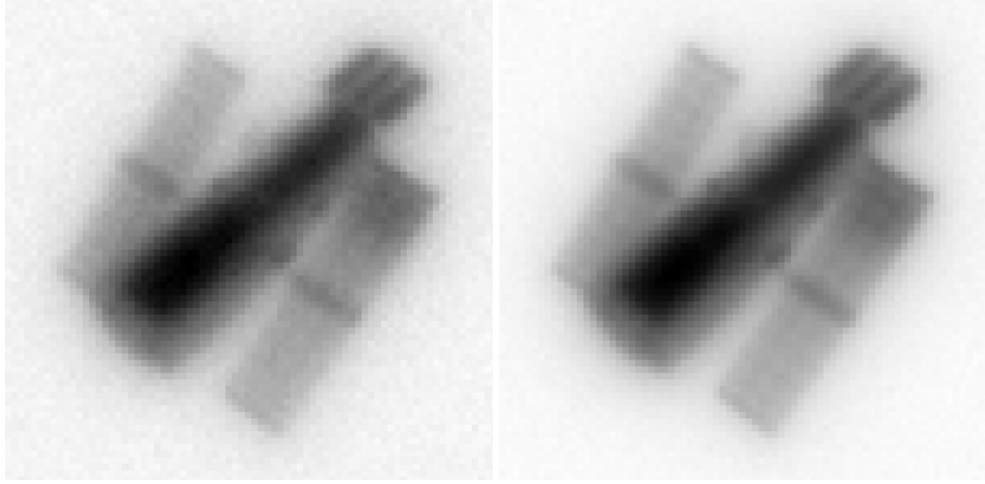


Fig. 5. Left: One of the 20 noisy frames used as input for MFBD. Right: integrated image used as input for MISTRAL. Inverted color scale.

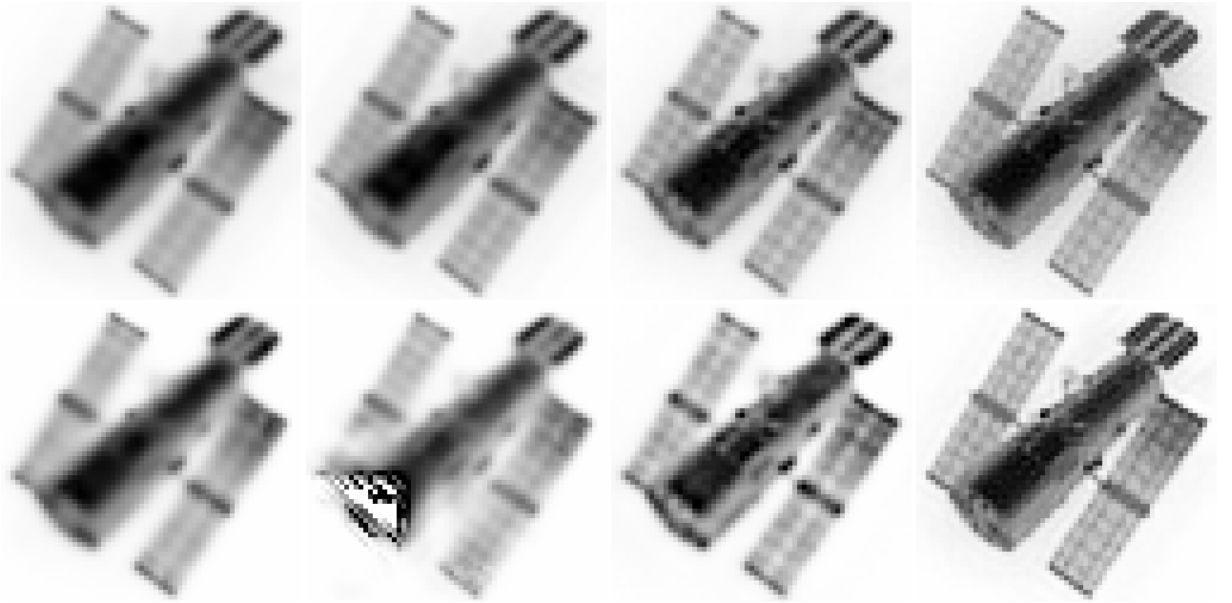


Fig. 6. Results of image reconstruction tests for the object illustrated in the rightmost panel in Fig. 1. From left to right: AWMLE (with wavelets), AWMLE (with curvelets), MFBD, MISTRAL. Top row: input PSF matches the data, bottom row: mismatched PSF.

The results confirm that myopic codes such as IDAC or MISTRAL perform better than static-PSF codes such as AWMLE even when the input PSF perfectly matches the observation. MISTRAL which is the maximum a posteriori code seems to be performing better than MFBF for this particular type of object and moderate AO PSF.

5. Conclusions

We have developed a theory of partially-developed, non-Gaussian speckle. The theory accounts for all the basic properties of speckle: possible spatial non-stationarity, the occurrence of “pinned” and “halo” speckles, and the peculiar behavior of the central point in the scenario of small phase aberrations. This new framework contains the classic Goodman model as the special case, and it is compatible with recently proposed models of the Strehl ratio variability in AO images. The theory has multiple applications. One that we want to explore in the near future is the possible utility of it for multi-frame blind deconvolution. We strongly believe that the theory could provide constraints which would guide the optimization process at the heart of blind deconvolution.

Publications resulting from this project:

Refereed

Gladysz, S., Yaitskova, N., Christou, J., *Statistics of Intensity in Adaptive-Optics Images and Their Usefulness for Detection and Photometry of Exoplanets*, Journal of the Optical Society of America, Feature Issue on Adaptive Optics, 27, A64, 2010

Conference proceedings

Gladysz, S., Baena Gallé, R., Christou, J., Roberts, L. C., Jr., *Differential Photometry in Adaptive Optics Imaging*, Proceedings of the AMOS Technical Conference, Maui, 14-17 September 2010, p. E24

Gladysz, S., Yaitskova, N., *Finding and Measuring Extrasolar Planets Using Speckle Statistics*, Proceedings of SPIE, Volume 7387, 73870Z-1, 2010

Gladysz, S., Yaitskova, N., Christou, J., *Novel, Multi-frame Approach to Photometry of Exoplanets*, Proceedings of SPIE, Volume 7736, 77361H-1, 2010

Yaitskova, N., Gladysz, S., *Telling Planets from Speckles Created by ELT Segmentation*, Proceedings of SPIE, Volume 7733, 773351-1, 2010

Gladysz, S., Martinez, P., Aller Carpentier, E., Christou, J. C., *Statistical Signal Enhancement in Adaptive-Optics Observations of Exoplanets*, in Adaptive Optics: Methods, Analysis and Applications, OSA 2009

Gladysz, S., Christou, J. C., *Differential Photometry through PDF Deconvolution*, in Adaptive Optics: Methods, Analysis and Applications, OSA 2009

Submitted

Baena Gallé, R., Gladysz, S., *Estimation of Differential Photometry in Adaptive Optics Observations with a Wavelet-based Maximum Likelihood Estimator*, revised version submitted to Publications of the Astronomical Society of the Pacific

Yaitskova, N., Gladysz, S., *First Order Speckle Statistics for Arbitrary Aberration Strength*, submitted to Journal of the Optical Society of America

Yaitskova, N., Gladysz, S., Gudimetla, R., *Exact Theory of Adaptive Optics Speckle and its Applications*, submitted to Adaptive Optics: Methods, Analysis and Applications, OSA 2011

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Oral conference presentations:

- Sept. 2010 - *Photometry after Adaptive Optics: Comparison of Approaches*, AMOS Technical Conference, Maui, Hawaii, USA
- Sept. 2010 - *Finding and Measuring Extrasolar Planets Using Speckle Statistics*, Speckle2010: “Speckle fields forever” conference, Florianopolis, Brazil
- June 2010 - *Novel, Multi-frame Approach to Photometry of Exoplanets*, SPIE Astronomical Telescopes and Instrumentation, San Diego, USA
- Oct. 2009 - *Telling Exoplanets from Speckles on ELTs*, Towards Other Earths: perspectives and limitations in the ELT era, Porto, Portugal
- Oct. 2009 - *Statistical Signal Enhancement in Adaptive-Optics Observations of Exoplanets*, Optical Society of America “Adaptive Optics: Methods, Analysis and Applications”, San Jose, USA
- Oct. 2009 - *Differential Photometry through PDF Deconvolution*, Optical Society of America “Adaptive Optics: Methods, Analysis and Applications”, San Jose, USA
- June 2009 - *Detection and Characterization of Extrasolar Planets with Multiple-frame, High-contrast Methods*, European Optical Society Topical Meeting on Advanced Imaging Techniques, Jena, Germany

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7. List of Symbols, Abbreviations, and Acronyms

AO – adaptive optics

AWMLE – adaptive wavelets maximum-likelihood estimator

MFBD – multi-frame blind deconvolution

OTF – optical transfer function

PDF – probability density function

PSF – point-spread function

SR – Strehl ratio